

# Financial Economic Theory and Engineering Formula Sheet

2012

Morning and afternoon exam booklets will include a formula package identical to the one attached to this study note. The exam committee felt that by providing many key formulas, candidates would be able to focus more of their exam preparation time on the application of the formulas and concepts to demonstrate their understanding of the syllabus material and less time on the memorization of the formulas.

The formula sheet was developed sequentially by reviewing the syllabus material for each major syllabus topic. Candidates should be able to follow the flow of the formula package easily. We recommend that candidates use the formula package concurrently with the syllabus material. Not every formula in the syllabus is in the formula package. **Candidates are responsible for all formulas on the syllabus, including those not on the formula sheet.**

Candidates should carefully observe the sometimes subtle differences in formulas and their application to slightly different situations. For example, there are several versions of the Black-Scholes-Merton option pricing formula to differentiate between instruments paying dividends, tied to an index, etc. Candidates will be expected to recognize the correct formula to apply in a specific situation of an exam question.

Candidates will note that the formula package does not generally provide names or definitions of the formula or symbols used in the formula. With the wide variety of references and authors of the syllabus, candidates should recognize that the letter conventions and use of symbols may vary from one part of the syllabus to another and thus from one formula to another.

We trust that you will find the inclusion of the formula package to be a valuable study aide that will allow for more of your preparation time to be spent on mastering the learning objectives and learning outcomes.

## General Formulas

**Lognormal distribution:**

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2} \frac{(\ln(x) - \mu)^2}{\sigma^2}\right\}$$
$$E[X] = e^{\mu + \sigma^2/2} \quad V[X] = e^{2\mu + \sigma^2} (e^{\sigma^2} - 1)$$
$$\Pr[X \leq x] = \Phi\left(\frac{\ln(x) - \mu}{\sigma}\right) \quad x > 0$$

**Normal distribution:**

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2} \frac{(x - \mu)^2}{\sigma^2}\right\}$$

**Capital Asset Pricing Model:**

$$E[R_i] = R_f + \beta_i (E[R_m] - R_f) \quad \text{where} \quad \beta_i = \frac{\text{Cov}(R_i, R_m)}{\text{Var}[R_m]}$$

**Weighted Average Cost of Capital:**

$$\text{WACC} = (1 - \tau_c)k_b \frac{B}{B + S} + k_s \frac{S}{B + S}$$

where  $k_b$ =return on debt,  $k_s$  =return in equity.

**GARCH(1,1) Model**

$$Y_t = \mu + \sigma_t \epsilon_t \quad \sigma_t^2 = \alpha_0 + \alpha_1(Y_{t-1} - \mu)^2 + \beta\sigma_{t-1}^2$$

**RSLN-2 Model**

$Y_t | \rho_t \sim N(\mu_{\rho_t}, \sigma_{\rho_t}^2)$  where  $\{\rho_t\}$ ,  $t = 1, 2, \dots$ , is a Markov process with 2-states.

$$p_{ij} = \Pr[\rho_{t+1} = j | \rho_t = i]$$

**Conditional Tail Expectation**

For loss  $L$ , continuous at  $V_\alpha = F_L^{-1}(\alpha)$ :

$$\text{CTE}_\alpha(L) = E[L | L > V_\alpha(L)]$$

If there is a probability mass at  $V_\alpha$ , define  $\beta' = \max\{\beta : V_\alpha = V_\beta\}$ , then

$$\text{CTE}_\alpha(L) = \frac{(1 - \beta')E[X | X > V_\alpha] + (\beta' - \alpha)V_\alpha}{1 - \alpha}$$

### Vasicek Model

$$\begin{aligned} dr &= a(b-r)dt + \sigma dz & P(t, T) &= A(t, T)e^{-B(t, T)r(t)} \\ B(t, T) &= \frac{1 - e^{-a(T-t)}}{a} & A(t, T) &= \exp \left[ \frac{(B(t, T) - T + t)(a^2b - \sigma^2/2)}{a^2} - \frac{\sigma^2 B(t, T)^2}{4a} \right] \end{aligned}$$

### Cox Ingersoll Ross Model

$$\begin{aligned} dr &= a(b-r)dt + \sigma\sqrt{r}dz & P(t, T) &= A(t, T)e^{-B(t, T)r(t)} \\ B(t, T) &= \frac{2(e^{\gamma(T-t)} - 1)}{(\gamma + a)(e^{\gamma(T-t)} - 1) + 2\gamma} & A(t, T) &= \left[ \frac{2\gamma e^{(a+\gamma)(T-t)/2}}{(\gamma + a)(e^{\gamma(T-t)} - 1) + 2\gamma} \right]^{2ab/\sigma^2} \end{aligned}$$

### Ho-Lee Model

$$\begin{aligned} dr &= \theta(t)dt + \sigma dz & \theta(t) &= F_t(0, t) + \sigma^2 t \\ P(t, T) &= A(t, T)e^{-r(t)(T-t)} \\ \ln A(t, T) &= \ln \frac{P(0, T)}{P(0, t)} + (T-t)F(0, t) - \frac{1}{2}\sigma^2 t(T-t)^2 \end{aligned}$$

### Hull-White (Extended Vasicek) Model

$$\begin{aligned} dr &= (\theta(t) - ar)dt + \sigma dz & \theta(t) &= F_t(0, t) + aF(0, t) + \frac{\sigma^2}{2a}(1 - e^{-2at}) \\ P(t, T) &= A(t, T)e^{-B(t, T)r(t)} \\ B(t, T) &= \frac{1 - e^{-a(T-t)}}{a} \\ \ln A(t, T) &= \ln \frac{P(0, T)}{P(0, t)} + B(t, T)F(0, t) - \frac{1}{4a^3}\sigma^2 (e^{-aT} - e^{-at})^2 (e^{2at} - 1) \end{aligned}$$

### Itô's Lemma

Let  $X$  be an Itô process such that  $dX_t = u(t, X_t) dt + v(t, X_t) dW_t$ ,

and let  $g(t, x)$  denote a twice differentiable function, then for  $Y_t = g(t, X_t)$

$$dY_t = \frac{\partial g(t, X_t)}{\partial t} dt + \frac{\partial g(t, X_t)}{\partial x} dX_t + \frac{1}{2} \frac{\partial^2 g(t, X_t)}{\partial x^2} (dX_t)^2.$$

### Geometric Brownian Motion

$$\frac{dS_t}{S_t} = \mu dt + \sigma dZ_t$$

### Black-Scholes Pricing Formulas

$$c_t = S_t N(d_1) - Ke^{-r(T-t)} N(d_2)$$

$$p_t = Ke^{-r(T-t)}N(-d_2) - S_tN(-d_1)$$

$$\text{where } d_1 = \frac{\ln(S_t/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \text{ and } d_2 = d_1 - \sigma\sqrt{T-t}$$

# Copeland, Weston, Shastri: Financial Theory and Corporate Policy

## Chapter 2

$$(2.2) \quad Rev_t + m_t S_t = Div_t + (W\&S)_t + I_t$$

$$(2.5) \quad NI_t = Rev_t - (W\&S)_t - dep_t$$

$$(2.7) \quad S_0 = \sum_{t=1}^{\infty} \frac{NI_t - \Delta A_t}{(1 + k_s)^t}$$

$$(2.13) \quad FCF = EBIT(1 - \tau_c) + \Delta dep - \Delta I$$

## Chapter 6

$$(6.34) \quad R_{jt} = E(R_{jt}) + \beta_j \delta_{mt} + \varepsilon_{jt}$$

$$(6.36) \quad R_{jt} - R_{ft} = (R_{mt} - R_{ft})\beta_j + \varepsilon_{jt}$$

$$(6.36) \quad R'_{pt} = \gamma_0 + \gamma_1 \beta_p + \varepsilon_{pt}$$

$$(6.37) \quad R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}$$

$$(6.38) \quad E(R_i) = E(R_Z) + [E(R_m) - E(R_Z)]\beta_i$$

$$(6.40) \quad E(R_i) - R_f = b_i [E(R_m) - R_f] + s_i E(\text{SMB}) + h_i E(\text{HML})$$

$$(6.41) \quad \lambda_i = E(R_m) - E(R_Z)$$

$$(6.46) \quad R_{it} = \hat{\gamma}_{0t} + \hat{\gamma}_{1t} \beta_{it} + \varepsilon_{it}$$

$$(6.49) \quad E(R_i) = E(R_{z,I}) + [E(R_I) - E(R_{z,I})]\beta_{i,I}$$

$$(6.50) \quad \tilde{R}_i = E(\tilde{R}_i) + b_{i1} \tilde{F}_1 + \dots + b_{ik} \tilde{F}_k + \tilde{\varepsilon}_i$$

$$(6.57) \quad E(\tilde{R}_i) = \lambda_0 + \lambda_1 b_{i1} + \dots + \lambda_k b_{ik}$$

$$(6.59) \quad E(R_i) - R_f = [\bar{\delta}_1 - R_f] b_{i1} + \dots + [\bar{\delta}_k - R_f] b_{ik}$$

$$(6.60) \quad b_{ik} = Cov(R_i, \delta_k) / Var(\delta_k)$$

## Chapter 9

$$(9.1) \quad C(S_A, S_B, T) = S_A N(d_1) - S_B N(d_2)$$

$$\text{where } d_1 = [\ln(S_A/S_B) + V^2 T] / V\sqrt{T}; \quad d_2 = d_1 - V\sqrt{T}$$

$$\text{and } V^2 = V_A^2 - 2\rho_{AB} V_A V_B + V_B^2$$

## Chapter 10

$$(10.1) \quad V(\eta) \equiv \sum_m q(m) \max_a \sum_e p(e|m) U(a, e) - V(\eta_0)$$

$$(10.4) \quad p(r - c_2) + (1 - p)(dr - c_2) = p(r/d - c_1) + (1 - p)(r - c_1)$$

$$(10.7) \quad \text{Fair Game: } \varepsilon_{j,t+1} = \frac{P_{j,t+1} - P_{jt}}{P_{jt}} - \frac{E(P_{j,t+1}|\eta_t) - P_{jt}}{P_{jt}}, \quad E(\varepsilon_{j,t+1}) = 0$$

Note: This is a correction to the formula in the reading.

$$(10.9a) \quad \text{Submartingale: } \frac{E(P_{j,t+1}|\eta_t) - P_{jt}}{P_{jt}} = E(r_{j,t+1}|\eta_t) > 0$$

$$(10.9b) \quad \text{Martingale: } \frac{E(P_{j,t+1}|\eta_t) - P_{jt}}{P_{jt}} = E(r_{j,t+1}|\eta_t) = 0$$

$$(10.14) \quad E(R_{jt}|\hat{\beta}_{jt}) = R_{jt} + \left[ E(R_{mt}|\hat{\beta}_{mt}) - R_{ft} \right] \hat{\beta}_{jt}$$

## Chapter 11

$$(11.2) \quad R_{jt} = \alpha_j + \beta_{1j}(R_{mt} - R_{ft}) + \beta_{2j}(RLE_t - RSE_t) + \beta_{3j}(HBTM_t - LBTM_t) + \varepsilon_{jt}$$

$$(11.3) \quad \Delta NI_{jt} = \hat{a} + \hat{b}_j \Delta m_t + \varepsilon_{jt}$$

$$(11.4) \quad \Delta \widehat{NI}_{j,t+1} = \hat{a} + \hat{b}_j \Delta m_{t+1}$$

## Chapter 12

$$(12.1) \quad V(\alpha) = \frac{1}{(1+r)} [\mu(\alpha) - \lambda]$$

$$(12.3) \quad W_0 = X + \beta V_M + Y - (1 - \alpha)V(\alpha)$$

$$(12.4) \quad \widetilde{W}_1 = \alpha[\mu + \tilde{\varepsilon} - \mu(\alpha) + \lambda] + \beta[\widetilde{M} - (1+r)V_M] + (1+r)(W_0 - X) + \mu(\alpha) - \lambda$$

$$(12.5) \quad \frac{\partial E(U(\widetilde{W}_1))}{\partial \alpha} = E \left[ U'(\widetilde{W}_1) (\mu + \tilde{\varepsilon} - \mu(\alpha) + \lambda + (1 - \alpha)\mu_\alpha) \right] = 0$$

$$(12.6) \quad \frac{\partial E(U(\widetilde{W}_1))}{\partial \beta} = E \left[ U'(\widetilde{W}_1) (\widetilde{M} - (1+r)V_M) \right] = 0 \quad \text{where} \quad \mu_\alpha = \frac{\partial \mu}{\partial \alpha}$$

$$(12.7) \quad (1 - \alpha)\mu_\alpha = - \frac{E \left[ U'(\widetilde{W}_1) (\tilde{\varepsilon} + \lambda) \right]}{E \left[ U'(\widetilde{W}_1) \right]}$$

$$(12.15) \quad E(D) = \frac{1}{1+r} \left[ V(D) + \mu - \tau_p D + \beta \int_{\underline{X}}^D (X - D) f(X) dX \right]$$

Note: This is a correction to the formula in the reading.

$$(12.16) \quad E(D) = \frac{1}{1+r} \left[ V(D) + \frac{t}{2} - \tau_p D - \beta \frac{D^2}{2t} \right]$$

$$(12.17) \quad V'(D^*) = \tau_p + \beta \frac{D^*}{t}$$

$$(12.18) \quad V[D^*(t)] = \frac{1}{r} \left[ \frac{t}{2} - \tau_p D^*(t) - \beta \frac{[D^*(t)]^2}{2t} \right]$$

$$(12.20) \quad V[D^*(t)] = (\tau_p + \beta A) D^*(t)$$

$$(12.21) \quad A = - \left[ \frac{\tau_p}{\beta} \right] \left[ \frac{1+r}{1+2r} \right] + \left[ \frac{\tau_p}{\beta} \right] \left[ \frac{1+r}{1+2r} \right] \sqrt{1 + \frac{\beta(1+2r)}{\tau_p^2(1+r)^2}}$$

$$(12.22) \quad I + D = C + Np_e = C + P_e$$

$$(12.24) \quad \max_D \left( L - \tau_p D + \left[ \frac{P + \tau_p D - L}{P + \tau_p D + I - C} \right] X \right)$$

$$(12.25) \quad \tau_p = \left( \tau_p + \frac{\partial P}{\partial D} \right) \frac{L + I - C}{P + \tau_p D + I - C} X$$

$$(12.26) \quad D(X) = \frac{1}{\tau_p} \max(I - C + L, 0) \ln X$$

$$(12.30) \quad V_0^{old} = \frac{P'}{P' + E} (E + S + a + b)$$

$$(12.36) \quad W^0(z) = \alpha \bar{P}(z) + \beta P - T(m, P)$$

$$(12.37) \quad W^s(n, z) = \alpha \hat{P}(n, z) + \beta P - T(n, p)$$

$$(12.38) \quad W_n = \alpha \hat{P}_n - \frac{t_2}{p^{\gamma-1}} = 0$$

$$(12.40) \quad \hat{P}(n, z) = k[n + c(z)]^{1/\gamma} \quad \text{where} \quad k = (t_2 \gamma / \alpha)^{1/\gamma}$$

$$(12.41) \quad M(n, z) = \hat{P}(n, z) - T(n, P) = k(1 - t_1)[n + c(z)]^{1/\gamma} - t_2 n k^{1-\gamma} [n + c(z)]^{(1-\gamma)/\gamma}$$

$$(12.45) \quad V_0 = (1 - t) X_0$$

$$(12.46) \quad E(X|n) = \hat{X}(n) = \frac{X_0 s_0 + \hat{Y}_m(n) s_m}{s_0 + s_m}$$

$$(12.47) \quad V_1(n) = \hat{X}(n) - T(n) - C$$

$$(12.50) \quad V_2(T, \bar{Y}) = \frac{X_0 s_0 + \hat{Y}_m s_m + \bar{Y} F T s}{s_0 + s_m + F T s} - E \left[ X t(X/n) | T, \hat{Y}_m, \bar{Y} \right] - C$$

$$(12.51) \quad E[V_2(T) | Y_m] = \frac{X_0 s_0 + \hat{Y}_m s_m + \left( \frac{X_0 s_0 + Y_m s_m}{s_0 + s_m} \right) F T s}{s_0 + s_m + F T s} - T - C$$

$$(12.54) \quad \left( \frac{d\alpha}{dD} \right) [V(n) - B(n, D)] - \alpha(D) \left( \frac{\partial B}{\partial D} \right) = 0$$

$$(12.55) \quad \left( \frac{d^2 \alpha}{dD^2} \right) [V(n) - B(n, D)] - 2 \left( \frac{d\alpha}{dD} \right) \left( \frac{\partial B}{\partial D} \right) - \alpha(D) \left( \frac{\partial^2 B}{\partial D^2} \right) < 0$$

$$(12.56) \quad \alpha(D^*(n)) [V(n) - B(n, D^*(n))] - [V(n) - I] = 0$$

$$(12.57) \quad \left[ \frac{d^2 \alpha}{dD^2} (V - B) - 2 \frac{d\alpha}{dD} \frac{\partial B}{\partial D} - \alpha \frac{\partial^2 B}{\partial D^2} \right] \frac{dD}{dn} = \frac{d\alpha}{dD} \left( \frac{\partial B}{\partial n} - \frac{dV}{dn} \right) + \alpha \left( \frac{\partial^2 B}{\partial n \partial D} \right)$$

$$(12.58) \quad \hat{\varepsilon}(\alpha, D) = \left[ (a - D_0)^2 + D^2 \frac{1 - \alpha}{\alpha^2} \right]$$

$$(12.59) \quad \bar{D} = X_1 \left[ \frac{\bar{\alpha}(\epsilon - \gamma)}{(1 - \bar{\alpha})(1 - \epsilon)\gamma} \right]$$

$$(12.60) \quad \max_{c(s,p),a} \int \int U [s - c(s, p)] f(s, p|a) ds dp$$

$$(12.61) \quad \int \int V [c(s, p)] f(s, p|a) ds dp - G(a) \geq \underline{V}$$

$$(12.63) \quad \frac{U'[s - c(s, p)]}{V'[c(s, p)]} = \lambda$$

$$(12.65) \quad \max_a U(k) + \lambda \left[ \int \int V [c(s, p)] f(s, p|a) ds dp - G(a) - \underline{V} \right]$$

$$(12.68) \quad \max_{c(s),a} \int U [s - c(s)] f(s|a) ds + \lambda \left[ \int V [c(s)] f(s|a) ds - G(a) - \underline{V} \right] \\ + \mu \left[ \int V [c(s)] f_a(s|a) ds - G'(a) \right]$$

$$(12.70) \quad \frac{U'[s - c(s)]}{V'[c(s)]} = \lambda + \mu \frac{f_a(s|a)}{f(s|a)}$$

$$(12.73) \quad \max_{c(s,p),a} \int \int U [s - c(s, p)] f(s, p|a) ds dp + \lambda \left[ \int \int V [c(s, p)] f(s, p|a) ds dp - G(a) - \underline{V} \right] \\ + \mu \left[ \int \int V [c(s, p)] f_a(s, p|a) ds dp - G'(a) \right]$$

$$(12.74) \quad \frac{U' [s - c(s, p)]}{V'[c(s, p)]} = \lambda + \mu \frac{f_a(s, p|a)}{f(s, p|a)}$$

$$(12.75) \quad \frac{U' [s - c(\mathbf{p})]}{V'[c(\mathbf{p})]} = \lambda + \mu_1 \frac{f_{a_1}(\mathbf{p}|a)}{f(\mathbf{p}|a)} + \mu_2 \frac{f_{a_2}(\mathbf{p}|a)}{f(\mathbf{p}|a)} + \dots + \mu_n \frac{f_{a_n}(\mathbf{p}|a)}{f(\mathbf{p}|a)}$$

$$(12.81) \quad \max_{c(s,\mathbf{p},m),a(m),m(m)} E_{s,\mathbf{p},m} [U [s - c(s, \mathbf{p}, m)] | a(m)]$$

Subject to (for all m)  $E_{s,\mathbf{p}|m} [[V[c(s, \mathbf{p}, m)] - G[a(m)]] | a(m)] \geq \underline{V}$

$$(12.82) \quad \frac{\partial V(X^*)}{\partial X} = \frac{\partial P(X^*)}{\partial X} - \frac{\partial C(X^*)}{\partial X} = 0$$

$$(12.83) \quad S = \frac{(1 - \beta)E(s) - \alpha - \lambda(1 - \beta)\text{Cov}(s, R_M)}{1 + r_f}$$

$$(12.86) \quad \max_{\alpha, \beta} \frac{(1 - \beta)E(s) - \alpha - \lambda(1 - \beta)\text{Cov}(s, R_M)}{1 + r_f}$$

$$+ \mu (a [E(W) + \alpha + \beta E(s)] - b [\text{Var}(W) + 2\beta\text{Cov}(W, s) + \beta^2\text{Var}(s)] - \underline{V})$$

$$(12.90) \quad \beta = \frac{\lambda a \text{Cov}(s, R_M)}{2b \text{Var}(s)} - \frac{\text{Cov}(W, s)}{\text{Var}(s)}$$



$$(12.94) \quad B_{new} - B = D \left\{ - \left[ P \left( \frac{V + dB}{D + dB}, 1, T, r_f, \sigma_V \right) \right] + P \left( \frac{V}{D}, 1, T, r_f, \sigma_V \right) \right\} = D(-P_X + P_Y)$$

## Chapter 15

$$(15.2) \quad V_U = \frac{E(\widetilde{FCF})}{\rho} \quad \text{or} \quad V_U = \frac{E(\widetilde{EBIT})(1 - \tau_c)}{\rho}$$

$$(15.3) \quad \widetilde{NI} + k_d D = \left( \widetilde{Rev} - \widetilde{VC} - FCC - dep \right) (1 - \tau_c) + k_d D \tau_c$$

$$(15.4) \quad V^L = \frac{E(\widetilde{EBIT})(1 - \tau_c)}{\rho} + \frac{k_d D \tau_c}{k_b}$$

$$(15.6) \quad V_L = V_U + \tau_c B$$

$$(15.9) \quad \frac{\Delta V_L}{\Delta I} = \frac{\Delta S^o}{\Delta I} + \frac{\Delta S^n + \Delta B^n}{\Delta I} = \frac{\Delta S^o}{\Delta I} + 1$$

$$(15.11) \quad \frac{(1 - \tau_c) \Delta E(EBIT)}{\Delta I} > \rho \left( 1 - \tau_c \frac{\Delta B}{\Delta I} \right)$$

$$(15.18) \quad k_s = \Delta NI / \Delta S = \rho + (1 - \tau_c)(\rho - k_b) \frac{\Delta B}{\Delta S}$$

$$(15.20) \quad G = V_L - V_U = \tau_c B$$

$$(15.21) \quad V_U = \frac{E(EBIT)(1 - \tau_c)(1 - \tau_{ps})}{\rho}$$

$$(15.23) \quad V_L = V_U + \left[ 1 - \frac{(1 - \tau_c)(1 - \tau_{ps})}{(1 - \tau_{pB})} \right] B \quad \text{where } B = k_d D(1 - \tau_{pB}) / k_b$$

$$(15.28) \quad \beta_L = \left[ 1 + (1 - \tau_c) \frac{B}{S} \right] \beta_U$$

$$(15.33) \quad R_f S^L + \lambda^*(1 - \tau_c) \text{Cov}(EBIT, R_m) - \lambda^*(1 - \tau_c) B [\text{Cov}(R_{bj}, R_m)] \\ = E(EBIT)(1 - \tau_c) - E(R_{bj})B(1 - \tau_c)$$

$$(15.38) \quad dS = \frac{\partial S}{\partial V} dV + \frac{\partial S}{\partial t} dt + \frac{1}{2} \frac{\partial^2 S}{\partial V^2} \sigma^2 V^2 dt$$

$$(15.40) \quad r_s = \frac{\partial S}{\partial V} \frac{V}{S} r_V$$

$$(15.43) \quad S = VN(d_1) - e^{-r_f T} DN(d_2)$$

$$(15.46) \quad \beta_S = \frac{1}{1 - (D/V)e^{-r_f T} [N(d_2)/N(d_1)]} \beta_V$$

$$(15.47/8) \quad k_s = R_f + (R_m - R_f)N(d_1) \frac{V}{S} \beta_V = R_f + N(d_1)(R_v - R_f) \frac{V}{S}$$

$$(15.52) \quad k_b = R_f + (\rho - R_f)N(-d_1) \frac{V}{B}$$

Note: This is a correction to the formula in the reading.

$$(15.55) \quad \frac{dV}{V} = \mu(V, t)dt + \sigma dW$$

$$(15.57) \quad \frac{1}{2}\sigma^2 V^2 F_{VV}(V) + rV F_V(V) - rF(V) + C = 0$$

$$(15.58) \quad F(V) = A_0 + A_1 V + A_2 V^{-(2r/\sigma^2)}$$

$$(15.59) \quad V = V_B \Rightarrow B(V) = (1 - \alpha)V_B \quad V \rightarrow \infty \Rightarrow B(V) \rightarrow C/r$$

$$(15.61) \quad B(V) = (1 - p_B)C/r + p_B[(1 - \alpha)V_B] \quad \text{where} \quad p_B = (V/V_B)^{-2r/\sigma^2}$$

$$(15.62) \quad V = V_B \Rightarrow DC(V) = \alpha V_B \quad V \rightarrow \infty \Rightarrow DC(V) \rightarrow 0$$

$$(15.63) \quad DC(V) = \alpha V_B (V/V_B)^{-2r/\sigma^2}$$

$$(15.67) \quad V_L(V) = V_U(V) + T_c B(V) - DC(V) = V_U(V) + T_c B - p_B T_c B - \alpha V_B p_B$$

$$(15.68) \quad M = (1 + r)\gamma_0 V_0 + \gamma_1 \begin{cases} V_1 & \text{if } V_1 \geq D \\ V_1 - C & \text{if } V_1 < D \end{cases}$$

$$(15.69) \quad M_a = \begin{cases} \gamma_0(1 + r)\frac{V_{1a}}{1+r} + \gamma_1 V_{1a} & \text{if } D^* < D \leq V_{1a} \\ \gamma_0(1 + r)\frac{V_{1b}}{1+r} + \gamma_1 V_{1a} & \text{if } D < D^* \end{cases}$$

$$(15.70) \quad M_b = \begin{cases} \gamma_0(1 + r)\frac{V_{1a}}{1+r} + \gamma_1(V_{1b} - C) & \text{if } D^* \leq D \leq V_{1a} \\ \gamma_0(1 + r)\frac{V_{1b}}{1+r} + \gamma_1 V_{1b} & \text{if } D < D^* \end{cases}$$

## Chapter 16

$$(16.2) \quad V_i(t) = \frac{Div_i(t+1) + n_i(t)P_i(t+1)}{1 + k_u(t+1)}$$

$$(16.8) \quad \tilde{V}_i(t) = \frac{\widetilde{EBIT}_i(t+1) - \tilde{I}_i(t+1) + \tilde{V}_i(t+1)}{1 + k_u(t+1)}$$

$$(16.9) \quad \tilde{Y}_{di} = \left[ \left( \widetilde{EBIT} - rD_c \right) (1 - \tau_c) - rD_{pi} \right] (1 - \tau_{pi})$$

$$(16.10) \quad \tilde{Y}_{gi} = \left( \widetilde{EBIT} - rD_c \right) (1 - \tau_c) (1 - \tau_{gi}) - rD_{pi} (1 - \tau_{pi})$$

$$(16.26) \quad S_1 - E(S_1) = \varepsilon_1 \left[ 1 + \frac{\gamma}{1+k} \right] = [EBIT_1 - E_0(EBIT_1)] \left[ 1 + \frac{\gamma}{1+k} \right]$$

$$(16.27) \quad \Delta Div_{it} = a_i + c_i(Div_{it}^* - Div_{i,t-1}) + U_{it}$$

$$(16.33) \quad \Delta Div_t = \beta_1 Div_{t-1} + \beta_2 NI_t + \beta_3 NI_{t-1} + Z_t$$

$$(16.35) \quad P_{it} = a + b Div_{it} + c RE_{it} + \varepsilon_{it}$$

$$(16.38) \quad \tilde{R}_j = \gamma_0 + \left[ \tilde{R}_m - \gamma_0 \right] \beta_j + \gamma_1 [DY_j - DY_m] / DY_m + \varepsilon_j$$

$$(16.43) \quad \frac{\Delta W}{N_0 P_0} = (1 - F_P) \left( \frac{P_E - P_0}{P_0} \right) + F_P \frac{P_T - P_0}{P_0}$$

# Hull: Options Futures and Other Derivatives

## Chapter 12: Binomial Trees

$$(12.1) \quad \Delta = \frac{f_u - f_d}{S_0u - S_0d}$$

$$(12.5/6) \quad f = e^{-r\Delta t} [pf_u + (1-p)f_d] \quad \text{where } p = \frac{e^{r\Delta t} - d}{u - d}$$

$$(12.13/14) \quad u = e^{\sigma\sqrt{\Delta t}} \quad d = e^{-\sigma\sqrt{\Delta t}}$$

## Chapter 13: Wiener Processes and Itô's Lemma

$$(13.17) \quad G = \ln S \quad dG = (\mu - \sigma^2/2) dt + \sigma dz$$

## Chapter 14: The Black-Scholes-Merton model

$$(14.9) \quad df = \left( \frac{\partial f}{\partial S} \mu S + \frac{\partial f}{\partial t} + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 \right) dt + \frac{\partial f}{\partial S} \sigma S dz$$

$$(14.16) \quad \frac{\partial f}{\partial t} + rS \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf$$

## Chapter 18: Greek Letters

$$(18. ) \quad \Delta(\text{call}) = N(d_1) \quad \Delta(\text{put}) = N(d_1) - 1$$

$$(18.2) \quad \Theta(\text{call}) = -\frac{S_0 N'(d_1) \sigma}{2\sqrt{T}} - rK e^{-rT} N(d_2)$$

$$(18.2) \quad \Theta(\text{put}) = -\frac{S_0 N'(d_1) \sigma}{2\sqrt{T}} + rK e^{-rT} N(-d_2)$$

$$(18. ) \quad \Gamma(\text{call}) = \Gamma(\text{put}) = \frac{N'(d_1)}{S_0 \sigma \sqrt{T}}$$

$$(18.4) \quad \Theta + rS\Delta + \frac{1}{2} \sigma^2 S^2 \Gamma = r\Pi$$

$$(18. ) \quad \mathcal{V}(\text{call}) = \mathcal{V}(\text{put}) = S_0 \sqrt{T} N'(d_1)$$

$$(18. ) \quad \text{rho}(\text{call}) = K T e^{-rT} N(d_2)$$

$$(18. ) \quad \text{rho}(\text{put}) = -K T e^{-rT} N(-d_2)$$

## Chapter 20: Basic Numerical Procedures

$$(20.8) \quad \Delta = \frac{f_{1,1} - f_{1,0}}{S_0u - S_0d}$$

$$(20.9) \quad \Gamma = \frac{(f_{2,2} - f_{2,1})/(S_0 u^2 - S_0) - (f_{2,1} - f_{2,0})/(S_0 - S_0 d^2)}{h}$$

$$(20.10) \quad \Theta = \frac{f_{2,1} - f_{0,0}}{2\Delta t}$$

$$(20.12) \quad p = \frac{e^{[f(t)-g(t)]\Delta t} - d}{u - d}$$

$$(20.27) \quad a_j f_{i,j-1} + b_j f_{i,j} + c_j f_{i,j+1} = f_{i+1,j}$$

$$\text{where } a_j = \frac{1}{2}(r - q)j\Delta t - \frac{1}{2}\sigma^2 j^2 \Delta t, \quad b_j = 1 + \sigma^2 j^2 \Delta t + r\Delta t$$

$$\text{and } c_j = -\frac{1}{2}(r - q)j\Delta t - \frac{1}{2}\sigma^2 j^2 \Delta t$$

$$(20.34) \quad f_{i,j} = a_j^* f_{i+1,j-1} + b_j^* f_{i+1,j} + c_j^* f_{i+1,j+1}$$

$$\text{where } a_j^* = \frac{1}{1 + r\Delta t} \left( -\frac{1}{2}(r - q)j\Delta t + \frac{1}{2}\sigma^2 j^2 \Delta t \right), \quad b_j^* = \frac{1}{1 + r\Delta t} (1 - \sigma^2 j^2 \Delta t)$$

$$\text{and } c_j^* = \frac{1}{1 + r\Delta t} \left( \frac{1}{2}(r - q)j\Delta t + \frac{1}{2}\sigma^2 j^2 \Delta t \right)$$

$$(20.35) \quad \alpha_j f_{i,j-1} + \beta_j f_{i,j} + \gamma_j f_{i,j+1} = f_{i+1,j}$$

$$\text{where } \alpha_j = \frac{\Delta t}{2\Delta Z} \left( r - q - \frac{\sigma^2}{2} \right) - \frac{\Delta t}{2\Delta Z^2} \sigma^2 \quad \beta_j = 1 + \frac{\Delta t}{\Delta Z^2} \sigma^2 + r\Delta t$$

$$\text{and } \gamma_j = \frac{-\Delta t}{2\Delta Z} \left( r - q - \frac{\sigma^2}{2} \right) - \frac{\Delta t}{2\Delta Z^2} \sigma^2$$

$$(20.36) \quad \alpha_j^* f_{i+1,j-1} + \beta_j^* f_{i+1,j} + \gamma_j^* f_{i+1,j+1} = f_{i,j}$$

$$\text{where } \alpha_j^* = \frac{1}{1 + r\Delta t} \left[ -\frac{\Delta t}{2\Delta Z} \left( r - q - \frac{\sigma^2}{2} \right) + \frac{\Delta t}{2\Delta Z^2} \sigma^2 \right]$$

$$\text{and } \beta_j^* = \frac{1}{1 + r\Delta t} \left( 1 - \frac{\Delta t}{\Delta Z^2} \sigma^2 \right)$$

$$\text{and } \gamma_j^* = \frac{1}{1 + r\Delta t} \left[ \frac{\Delta t}{2\Delta Z} \left( r - q - \frac{\sigma^2}{2} \right) + \frac{\Delta t}{2\Delta Z^2} \sigma^2 \right]$$

## Chapter 22: Estimating Volatilities and Correlations

$$(22.7) \quad \sigma_n^2 = \lambda \sigma_{n-1}^2 + (1 - \lambda) u_{n-1}^2$$

$$(22.8) \quad \sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$

$$(22.12) \quad \sum_{i=1}^m \left[ -\ln(v_i) - \frac{u_i^2}{v_i} \right]$$

$$(22.13) \quad E[\sigma_{n+t}^2] = V_L + (\alpha + \beta)^t (\sigma_n^2 - V_L)$$

## Chapter 25: Exotic Options

Call on a call

$$S_0 e^{-qT_2} M\left(a_1, b_1; \sqrt{T_1/T_2}\right) - K_2 e^{-rT_2} M\left(a_2, b_2; \sqrt{T_1/T_2}\right) - e^{-rT_1} K_1 N(a_2)$$

$$\text{where } a_1 = \frac{\ln(S_0/S^*) + (r - q + \sigma^2/2)T_1}{\sigma\sqrt{T_1}} \quad a_2 = a_1 - \sigma\sqrt{T_1}$$

$$b_1 = \frac{\ln(S_0/K_2) + (r - q + \sigma^2/2)T_2}{\sigma\sqrt{T_2}} \quad b_2 = b_1 - \sigma\sqrt{T_2}$$

Put on a call

$$K_2 e^{-rT_2} M\left(-a_2, b_2; -\sqrt{T_1/T_2}\right) - S_0 e^{-qT_2} M\left(-a_1, b_1; -\sqrt{T_1/T_2}\right) + e^{-rT_1} K_1 N(-a_2)$$

Call on a put

$$K_2 e^{-rT_2} M\left(-a_2, -b_2; \sqrt{T_1/T_2}\right) - S_0 e^{-qT_2} M\left(-a_1, -b_1; \sqrt{T_1/T_2}\right) - e^{-rT_1} K_1 N(-a_2)$$

Put on a put

$$S_0 e^{-qT_2} M\left(a_1, -b_1; -\sqrt{T_1/T_2}\right) - K_2 e^{-rT_2} M\left(a_2, -b_2; -\sqrt{T_1/T_2}\right) + e^{-rT_1} K_1 N(a_2)$$

Chooser

$$\max(c, p) = c + e^{-q(T_2-T_1)} \max\left(0, K e^{-(r-q)(T_2-T_1)} - S_1\right)$$

Barrier Options

$$\lambda = \frac{r - q + \sigma^2/2}{\sigma^2} \quad y = \frac{\ln[H^2/(S_0K)]}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

$$x_1 = \frac{\ln(S_0/H)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T} \quad y_1 = \frac{\ln(H/S_0)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

If  $H \leq K$

$$c_{di} = S_0 e^{-qT} (H/S_0)^{2\lambda} N(y) - K e^{-rT} (H/S_0)^{2\lambda-2} N\left(y - \sigma\sqrt{T}\right)$$

If  $H \geq K$

$$c_{do} = S_0 N(x_1) e^{-qT} - K e^{-rT} N\left(x_1 - \sigma\sqrt{T}\right) \\ - S_0 e^{-qT} (H/S_0)^{2\lambda} N(y_1) + K e^{-rT} (H/S_0)^{2\lambda-2} N\left(y_1 - \sigma\sqrt{T}\right)$$

If  $H > K$

$$c_{ui} = S_0 N(x_1) e^{-qT} - K e^{-rT} N\left(x_1 - \sigma\sqrt{T}\right) \\ - S_0 e^{-qT} (H/S_0)^{2\lambda} [N(-y) - N(-y_1)] \\ + K e^{-rT} (H/S_0)^{2\lambda-2} \left[ N\left(-y + \sigma\sqrt{T}\right) - N\left(-y_1 + \sigma\sqrt{T}\right) \right]$$

If  $H \geq K$

$$p_{ui} = -S_0 e^{-qT} (H/S_0)^{2\lambda} N(-y) + K e^{-rT} (H/S_0)^{2\lambda-2} N\left(-y + \sigma\sqrt{T}\right)$$

If  $H \leq K$

$$p_{uo} = -S_0 N(-x_1) e^{-qT} + K e^{-rT} N\left(-x_1 + \sigma\sqrt{T}\right) \\ + S_0 e^{-qT} (H/S_0)^{2\lambda} N(-y_1) - K e^{-rT} (H/S_0)^{2\lambda-2} N\left(-y_1 + \sigma\sqrt{T}\right)$$

If  $H \leq K$

$$p_{di} = -S_0 N(-x_1) e^{-qT} + K e^{-rT} N\left(-x_1 + \sigma\sqrt{T}\right) + S_0 e^{-qT} (H/S_0)^{2\lambda} [N(y) - N(y_1)] \\ - K e^{-rT} (H/S_0)^{2\lambda-2} \left[ N\left(y - \sigma\sqrt{T}\right) - N\left(y_1 - \sigma\sqrt{T}\right) \right]$$

Lookback Options

$$c_{fl} = S_0 e^{-qT} N(a_1) - S_0 e^{-qT} \frac{\sigma^2}{2(r-q)} N(-a_1) - S_{\min} e^{-rT} \left( N(a_2) - \frac{\sigma^2}{2(r-q)} e^{Y_1} N(-a_3) \right)$$

$$\text{where } a_1 = \frac{\ln(S_0/S_{\min}) + (r-q+\sigma^2/2)T}{\sigma\sqrt{T}} \quad a_2 = a_1 - \sigma\sqrt{T} \\ a_3 = \frac{\ln(S_0/S_{\min}) + (-r+q+\sigma^2/2)T}{\sigma\sqrt{T}} \\ Y_1 = \frac{2(r-q-\sigma^2/2)\ln(S_0/S_{\min})}{\sigma^2}$$

$$p_{fl} = S_{\max} e^{-rT} \left( N(b_1) - \frac{\sigma^2}{2(r-q)} e^{Y_2} N(-b_3) \right) + S_0 e^{-qT} \frac{\sigma^2}{2(r-q)} N(-b_2) - S_0 e^{-qT} N(b_2)$$

$$\text{where } b_1 = \frac{\ln(S_{\max}/S_0) + (-r+q+\sigma^2/2)T}{\sigma\sqrt{T}} \\ b_2 = b_1 - \sigma\sqrt{T} \\ b_3 = \frac{\ln(S_{\max}/S_0) + (r-q-\sigma^2/2)T}{\sigma\sqrt{T}} \\ Y_2 = \frac{2(r-q-\sigma^2/2)\ln(S_{\max}/S_0)}{\sigma^2}$$

Exchange Options

$$(25.5) \quad V_0 e^{-q_V T} N(d_1) - U_0 e^{-q_U T} N(d_2)$$

$$\text{where } d_1 = \frac{\ln(V_0/U_0) + (q_U - q_V + \hat{\sigma}^2/2)T}{\hat{\sigma}\sqrt{T}} \quad d_2 = d_1 - \hat{\sigma}\sqrt{T}$$

$$\text{and } \hat{\sigma} = \sqrt{\sigma_U^2 + \sigma_V^2 - 2\rho\sigma_U\sigma_V}$$

$$\text{Realized volatility: } \bar{\sigma} = \sqrt{\frac{252}{n-2} \sum_{i=1}^{n-1} \left[ \ln\left(\frac{S_{i+1}}{S_i}\right) \right]^2}$$

$$(25.6) \quad \hat{E}(\bar{V}) = \frac{2}{T} \ln \frac{F_0}{S^*} - \frac{2}{T} \left[ \frac{F_0}{S^*} - 1 \right] + \frac{2}{T} \left[ \int_{K=0}^{S^*} \frac{1}{K^2} e^{rT} p(K) dK + \int_{K=S^*}^{\infty} \frac{1}{K^2} e^{rT} c(K) dK \right]$$

$$(25.7) \quad L_{var}[\hat{E}(\bar{V}) - V_K]e^{-rT}$$

$$(25.8) \quad \int_{K=0}^{S^*} \frac{1}{K^2} e^{rT} p(K) dK + \int_{K=S^*}^{\infty} \frac{1}{K^2} e^{rT} c(K) dK = \sum_{i=1}^n \frac{\Delta K_i}{K_i^2} e^{rT} Q(K_i)$$

$$(25.9) \quad \hat{E}(\bar{\sigma}) = \sqrt{\hat{E}(\bar{V})} \left\{ 1 - \frac{1}{8} \left[ \frac{var(\bar{V})}{\hat{E}(\bar{V})^2} \right] \right\}$$

$$(25.10) \quad \hat{E}(\bar{V})T = - \left( \frac{F_0}{S^*} - 1 \right)^2 + 2 \sum_{i=1}^n \frac{\Delta K_i}{K_i^2} e^{rT} Q(K_i)$$

## Chapter 26: More on Models and Numerical Procedures

$$(26.2, 3) \quad \frac{dS}{S} = (r - q)dt + \sqrt{V} dz_S \quad dV = a(V_L - V)dt + \xi V^\alpha dz_V$$

## Chapter 27: Martingales and Measures

$$(27.4) \quad \Pi = (\sigma_2 f_2) f_1 - (\sigma_1 f_1) f_2$$

$$(27.5) \quad \Delta \Pi = (\mu_1 \sigma_2 f_1 f_2 - \mu_2 \sigma_1 f_1 f_2) \Delta t$$

$$(27.7) \quad \frac{df}{f} = \mu dt + \sigma dz$$

$$(27.8) \quad \frac{\mu - r}{\sigma} = \lambda$$

$$(27.13) \quad \mu - r = \sum_{i=1}^n \lambda_i \sigma_i$$

$$(27.14) \quad d \left( \frac{f}{g} \right) = (\sigma_f - \sigma_g) \frac{f}{g} dz$$

$$(27.20) \quad f_0 = P(0, T) E_T(f_T)$$

$$(27.23) \quad s(t) = \frac{P(t, T_0) - P(t, T_N)}{A(t)}$$

$$(27.25) \quad f_0 = A(0) E_A \left[ \frac{f_T}{A(T)} \right]$$

$$(27.26) \quad c = P(0, T) E_T [\max(S_T - K, 0)]$$

$$(27.31) \quad f_0 = U_0 E_U \left[ \max \left( \frac{V_T}{U_T} - 1, 0 \right) \right]$$

$$(27.32) \quad f_0 = V_0 N(d_1) - U_0 N(d_2)$$

$$(27.35) \quad \alpha_v = \rho \sigma_v \sigma_w$$

## Chapter 28: Interest Rate Derivatives: The Standard Market Models

$$(28.1) \quad c = P(0, T) [F_B N(d_1) - KN(d_2)]$$

$$(28.2) \quad p = P(0, T) [KN(-d_2) - F_B N(-d_1)]$$

$$\text{where } d_1 = \frac{\ln(F_B/K) + \sigma_B^2 T/2}{\sigma_B \sqrt{T}} \quad d_2 = d_1 - \sigma_B \sqrt{T}$$

$$(28.3) \quad F_B = \frac{B_0 - I}{P(0, T)}$$

$$(28.4) \quad \sigma_B = Dy_0 \sigma_y$$

$$(28.7, \text{ Caplet}) \quad L\delta_k P(0, t_{k+1}) [F_k N(d_1) - R_K N(d_2)]$$

$$\text{where } d_1 = \frac{\ln(F_k/R_K) + \sigma_k^2 t_k/2}{\sigma_k \sqrt{t_k}} \quad d_2 = d_1 - \sigma_k \sqrt{t_k}$$

$$(28.8, \text{ Floorlet}) \quad L\delta_k P(0, t_{k+1}) [R_K N(-d_2) - F_k N(-d_1)]$$

$$(28.10) \quad LA [s_0 N(d_1) - s_k N(d_2)] \quad \text{where } A = \frac{1}{m} \sum_{i=1}^{mn} P(0, T_i)$$

## Chapter 29: Convexity, Timing, and Quanto Adjustments

$$(29.1) \quad E_T(y_T) = y_0 - \frac{1}{2} y_0^2 \sigma_y^2 T \frac{G''(y_0)}{G'(y_0)}$$

$$(29.2) \quad E_T(R_T) = R_0 + \frac{R_0^2 \sigma_R^2 \tau T}{1 + R_0 \tau}$$

$$(29.3) \quad \alpha_V = \rho_{VW} \sigma_V \sigma_W$$

$$(29.4) \quad E_{T^*}(V_T) = E_T(V_T) \exp \left[ -\frac{\rho_{VR} \sigma_V \sigma_R R_0 (T^* - T)}{1 + R_0/m} T \right]$$

## Chapter 30: Interest Rate Derivatives: Models of the Short Rate

$$(30.20) \quad c = LP(0, s)N(h) - KP(0, T)N(h - \sigma_p)$$

$$p = KP(0, T)N(-h + \sigma_p) - LP(0, s)N(-h)$$

$$h = \frac{1}{\sigma_p} \ln \frac{LP(0, s)}{P(0, T)K} + \frac{\sigma_p}{2}$$

$$\sigma_p = \frac{\sigma}{a} [1 - e^{-a(s-T)}] \sqrt{\frac{1 - e^{-2aT}}{2a}}$$

$$(30.24) \quad P_{m+1} = \sum_{j=-n_m}^{n_m} Q_{m,j} \exp [-g(\alpha_m + j\Delta x) \Delta t]$$



## Chapter 32: Swaps Revisited - This Chapter is no longer on the syllabus

$$(32.1) \quad F_i + \frac{F_i^2 \sigma_i^2 \tau_i t_i}{1 + F_i \tau_i}$$

$$(31.2) \quad y_i - \frac{1}{2} y_i^2 \sigma_{y,i}^2 t_i \frac{G_i''(y_i)}{G_i'(y_i)} - \frac{y_i \tau_i F_i \rho_i \sigma_{y,i} \sigma_{F,i} t_i}{1 + F_i \tau_i}$$

$$(32.3) \quad V_i + V_i \rho_i \sigma_{W,i} \sigma_{V,i} t_i$$

## Hardy: Investment Guarantees

### Chapter 2

AR(1) (2.7):

$$(2.7) \quad Y_t = \mu + a(Y_{t-1} - \mu) + \sigma \varepsilon_t$$

$\varepsilon_t$  independent and identically distributed (iid),  $\varepsilon_t \sim N(0, 1)$

ARCH(1) (2.8-9):

$$(2.8) \quad Y_t = \mu + \sigma_t \varepsilon_t$$

$$(2.9) \quad \sigma_t^2 = a_0 + a_1 (Y_{t-1} - \mu)^2$$

ARCH(1) with AR(1) stock price (2.10-11):

$$(2.10) \quad Y_t = \mu + a(Y_{t-1} - \mu) + \sigma_t \varepsilon_t$$

$\varepsilon_t$  iid  $\sim N(0, 1)$

$$(2.11) \quad \sigma_t^2 = a_0 + \alpha (Y_{t-1} - \mu)^2$$

GARCH(1,1) (2.12-13):

$$(2.12) \quad Y_t = \mu + \sigma_t \varepsilon_t$$

$$(2.13) \quad \sigma_t^2 = a_0 + \alpha_1 (Y_{t-1} - \mu)^2 + \beta \sigma_{t-1}^2$$

GARCH(1,1) with AR(1) stock price (2.14-15):

$$(2.14) \quad Y_t = \mu + a(Y_{t-1} - \mu) + \sigma_t \varepsilon_t \quad \varepsilon_t \text{ iid } \sim N(0, 1)$$

$$(2.15) \quad \sigma_t^2 = \alpha_0 + \alpha_1 (Y_{t-1} - \mu)^2 + \beta \sigma_{t-1}^2$$

### Chapter 3

$$(3.1) \quad L(\theta) = f(X_1, X_2, \dots, X_n; \theta)$$

$$(3.2) \quad L(\theta) = \prod_{t=1}^n f(x_t; \theta) \quad l(\theta) = \sum_{t=1}^n \log f(x_t; \theta)$$

$$(3.3) \quad L(\theta) = f(x_1; \theta) f(x_2; \theta | x_1) f(x_3; \theta | x_1, x_2) \cdots f(x_n; \theta | x_1, \dots, x_{n-1})$$

$$(3.4) \quad l(\theta) = \sum_{t=1}^n \log f(x_t; \theta | x_1, \dots, x_{t-1})$$

$$(3.5) \quad E[Y_t] = \mu \quad \text{for all } t$$

$$(3.6) \quad E[(Y_t - \mu)(Y_{t-j} - \mu)] = \gamma_j \quad \text{for all } t \text{ and } j$$

$$(3.7) \quad b(\theta) = E[\hat{\theta} - \theta]$$

Lognormal model (3.13-14, 22-23):

$$(3.13) \quad \hat{\mu} = \bar{y}$$

$$(3.14) \quad \hat{\sigma} = \sqrt{\frac{\sum_{t=1}^n (y_t - \hat{\mu})^2}{n}}$$

$$(3.22) \quad \Sigma = \begin{pmatrix} \sigma^2/n & 0 \\ 0 & \sigma^2/2n \end{pmatrix}$$

$$(3.23) \quad \Sigma \approx \begin{pmatrix} \hat{\sigma}^2/n & 0 \\ 0 & \hat{\sigma}^2/2n \end{pmatrix}$$

AR(1) model (3.24-27):

$$(3.24) \quad Y_t | Y_{t-1} \sim N(\mu(1-a) + aY_{t-1}, \sigma^2) \quad t = 2, 3, \dots, n$$

$$(3.25, 26) \quad \begin{aligned} l(\mu, \sigma, a) &= \log \left( \sqrt{\frac{1-a^2}{2\pi\sigma^2}} \exp \left\{ -\frac{1}{2} \left( \frac{(Y_1 - \mu)^2(1-a^2)}{\sigma^2} \right) \right\} \right) \\ &\quad + \sum_{t=2}^n \log \left( \sqrt{\frac{1}{2\pi\sigma^2}} \exp \left\{ -\frac{1}{2} \left( \frac{(Y_t - (1-a)\mu - aY_{t-1})^2}{\sigma^2} \right) \right\} \right) \\ &= -\frac{n}{2} \log(2\pi) + \frac{1}{2} \log(1-a^2) - n \log \sigma \\ &\quad - \frac{1}{2} \left\{ \left( \frac{(Y_1 - \mu)^2(1-a^2)}{\sigma^2} \right) + \sum_{t=2}^n \left( \frac{(Y_t - (1-a)\mu - aY_{t-1})^2}{\sigma^2} \right) \right\} \end{aligned}$$

$$(3.27) \quad V[\hat{\mu}] \approx \frac{\sigma^2}{n(1-a^2)} \quad V[\hat{\sigma}] \approx \frac{\sigma^2}{2n} \quad V[\hat{a}] \approx \frac{1-a^2}{n}$$

ARCH(1) (3.28-30):

$$(3.28) \quad Y_t = \mu + \sigma_t \varepsilon_t$$

$$(3.29) \quad \sigma_t^2 = a_0 + a_1(Y_{t-1} - \mu)^2$$

$$(3.30) \quad Y_t | Y_{t-1} \sim N(\mu, a_0 + a_1(Y_{t-1} - \mu)^2) \quad t = 2, 3, \dots, n$$

GARCH(1,1) (3.31-33):

$$(3.31) \quad Y_t = \mu + \sigma_t \varepsilon_t$$

$$(3.32) \quad \sigma_t^2 = \alpha_0 + \alpha_1(Y_{t-1} - \mu)^2 + \beta\sigma_{t-1}^2$$

$$(3.33) \quad Y_t|Y_{t-1} \sim N(\mu, \sigma_t^2) \quad t = 2, 3, \dots, n$$

$$(3.35) \quad \tilde{\mu} = \bar{y} \quad \tilde{\sigma} = s_y$$

## Chapter 8

$$(8. ) \quad F_T = F_0 \frac{S_T}{S_0} (1 - m)^T$$

$$(8.3) \quad P_0 = Ge^{-rT} \Phi(-d_2) - S_0(1 - m)^T \Phi(-d_1)$$

$$\text{where } d_1 = \frac{\log(S_0(1 - m)^T/G) + (r + \sigma^2/2)T}{(\sigma\sqrt{T})} \quad d_2 = d_1 - \sigma\sqrt{T}$$

$$(8.8) \quad H(0) = \int_0^n (Ge^{-rt} \Phi(-d_2)) {}_t p_x^\tau \mu_{x,t}^{(d)} dt + \int_0^n (-S_0(1 - m)^t \Phi(-d_1)) {}_t p_x^\tau \mu_{x,t}^{(d)} dt$$

$$(8.9) \quad H(0, t_3) = (P_S(t_1) + S_0(1 - m)^{t_1}) \times \\ \{1 + P(t_2 - t_1)(1 + P(t_3 - t_2)) + (1 - m)^{t_2 - t_1} P(t_3 - t_2)\} \\ - S_0(1 - m)^{t_1}$$

$$\text{where } P_S(t_1) = \text{BSP}(S_0(1 - m)^{t_1}, G, t_1)$$

$$(8.15) \quad \alpha = \frac{B}{S_0 \ddot{a}_{x:\overline{n}|}^{\tau}}$$

$$(8.19) \quad \text{HE}_t = H(t) + {}_{t-1}q_x^d ((G - F_t)^+) - H(t^-)$$

$$(8.22) \quad \text{TC}_t = \tau S_t |\Psi_t - \Psi_{t-1}|$$

## FET-108-07 and FET-165-08: Doherty; Integrated Risk Management

### Chapter 13

$$(p. 465) \quad V(E) = V(F) - V(D) \\ = V(F) - D_{DF} + P(V(F), D) \\ = C(V(F), D)$$

$$(p. 481) \quad V^*(F) = S + D \left(1 + \frac{m}{n}\right)$$

$$(p. 494) \quad V'_R(E) = -C + V_R(F) - D + P\{V_r(F), D\} \\ = -C + V_R - D + P_R \\ V'_N(E) = V_N(F) - D + P\{V_N(F), D\} \\ = V_N - D + P_N$$

## Chapter 16

$$(16.1) \quad T = (E + P)(1 + r_i) - L$$

$$(16.2) \quad E(T) = (E + P - R(I; S))(1 + E(r_i)) - E(L(a)) + hC(I; S) - a$$

$$(16.3) \quad \frac{\partial E(T)}{\partial a} = \frac{-\partial E(L(a))}{\partial a} - 1 + h \frac{\partial C}{\partial I} \frac{\partial I}{\partial L} \frac{\partial L}{\partial a} = 0$$

### Manistre and Hancock; Variance of the CTE Estimator

$$(p. 130) \quad C\hat{T}E(\alpha) = \frac{1}{k} \sum_{j=1}^k x_{(j)}$$

$$(p. 131) \quad VAR(C\hat{T}E) = E[VAR(C\hat{T}E|X_{(k)})] + VAR[E(C\hat{T}E|X_{(k)})]$$

$$(p. 131) \quad VAR(C\hat{T}E) \approx \frac{VAR(x_{(1)}, \dots, x_{(k)}) + \alpha \cdot (C\hat{T}E - x_{(k)})^2}{k}$$

$$(p. 132) \quad C\hat{T}E(\alpha) = \frac{1}{k} \sum_{i=1}^k x_{(i)} \quad V\hat{a}R(\alpha) = x_{(k)}$$

$$(p. 133) \quad IF_{VaR}(x) = \begin{cases} \frac{-(1-\alpha)}{f(VaR)} & x < VaR \\ 0 & x = VaR \\ \frac{\alpha}{f(VaR)} & x > VaR \end{cases}$$

$$(p. 133) \quad IF_{CTE}(x) = \begin{cases} VaR - CTE & x < VaR \\ VaR - CTE + \frac{x - VaR}{1-\alpha} & x > VaR \end{cases}$$

$$(p. 133) \quad VAR(C\hat{T}E_n) \approx \frac{E[IF_{CTE}^2]}{n} = \frac{VAR(X|X > VaR) + \alpha \cdot (CTE - VaR)^2}{n \cdot (1 - \alpha)}$$

$$(p. 133) \quad VAR(V\hat{a}R_n) \approx \frac{E[IF_{VaR}^2]}{n} = \frac{\alpha \cdot (1 - \alpha)}{n \cdot [f(VaR)]^2}$$

$$(p. 133) \quad Cov(C\hat{T}E_n, V\hat{a}R_n) \approx \frac{E[IF_{CTE} \cdot IF_{VaR}]}{n} = \frac{\alpha \cdot (CTE - VaR)}{n \cdot f(VaR)}$$

$$(p. 134) \quad FSE(CTE) = \sqrt{\frac{VAR(X_{(1)}, \dots, X_{(k)}) + \alpha \cdot (C\hat{T}E - X_{(k)})^2}{n \cdot (1 - \alpha)}}$$

$$(p. 134) \quad FSE(VaR) = \frac{1}{\hat{f}(VaR)} \cdot \sqrt{\frac{\alpha \cdot (1 - \alpha)}{n}}$$

$$(p. 134) \quad Cov(CTE, VaR) = \frac{\alpha \cdot (CTE - X_{(k)})}{n \cdot \hat{f}(VaR)}$$

$$(p. 146) \quad VAR(V\hat{a}R_n) \approx \frac{E_G[(IF_{VaR})^2]}{n} = \frac{VAR_G[WH]}{n \cdot [f(VaR)]^2}$$

$$(p. 146) \quad VAR(C\hat{T}E_n) \approx \frac{E_G[(IF_{CTE})^2]}{n} = \frac{VAR_G[W(X - VaR)H]}{n \cdot (1 - \alpha)^2}$$

$$(p. 146) \quad Cov(C\hat{T}E_n, V\hat{a}R_n) \approx \frac{E_G[IF_{CTE} \cdot IF_{VaR}]}{n} = \frac{Cov_G[WH, WH(X - VaR)]}{n \cdot f(VaR) \cdot (1 - \alpha)}$$

$$(p. 146) \quad VAR(V\hat{a}R_n) \approx (1 - \alpha) \cdot \frac{E_F[W|X \geq VaR] - 1 + \alpha}{n \cdot [f(VaR)]^2}$$

$$(p. 146) \quad \begin{aligned} VAR(C\hat{T}E_n) &\approx \frac{VAR_F[\sqrt{WH}(X - VaR)]}{n \cdot (1 - \alpha)^2} \\ &= \frac{E_F[W|X \geq VaR] \cdot VAR_F[X|X \geq VaR]}{n \cdot (1 - \alpha)} \\ &\quad + \frac{(CTE - VaR)^2 \cdot (E_F[W|X \geq VaR] - 1 + \alpha)}{n \cdot (1 - \alpha)} \\ &\quad + \frac{Cov_F[W, (X - VaR)^2|X \geq VaR]}{n \cdot (1 - \alpha)} \end{aligned}$$

$$(p. 146) \quad \begin{aligned} Cov(C\hat{T}E, V\hat{a}R) &\approx \frac{Cov_F[W, X - VaR|X \geq VaR]}{n \cdot f(VaR)} \\ &\quad + \frac{1 - \alpha}{n \cdot f(VaR)} \cdot \{E_F[W|X \geq VaR] - 1 + \alpha\} \end{aligned}$$

**FET-106-07: Ho and Lee, The Oxford Guide to Financial Modeling**  
**Chapter 5: Interest Rate Derivatives: Interest Rate Models**

$$(5.10) \quad \begin{aligned} dr &= a_1 + b_1(l - r)dt + r\sigma_1 dZ \\ dl &= (a_2 + b_2r + c_2l) dt + l\sigma_2 dW \end{aligned}$$

$$(5.12) \quad \begin{aligned} dV &= M(t, r)dt + \Omega(t, r)dZ \\ M(t, r) &= V_t + \mu(t, r)V_r + \frac{1}{2}\sigma(t, r)^2V_{rr} \\ \Omega(t, r) &= \sigma(t, r)V_r \end{aligned}$$

**Chapter 6: Implied Volatility Surface: Calibrating the Models**

$$(6.10) \quad \begin{aligned} L(k, j + 1) \\ = L(k, j) \exp \left[ \left( \sum_{i=j+1}^k \frac{L(i, j)\Delta}{1+L(i, j)\Delta} \Lambda_{i-j-1} \Lambda_{k-j-1} - \frac{\Lambda_{k-j-1}^2}{2} \right) \Delta + \Delta_{k-j-1} \sqrt{\Delta} Z \right] \end{aligned}$$

$$(6.13) \quad P(T^*, i; T) = \frac{P(T^* + T)}{P(T^*)} 2 \frac{\prod_{t=T}^{T+T^*-1} h(t)}{\prod_{t=1}^{T^*-1} h(t)} \delta^{Ti} \quad \text{where } h(t) = \frac{1}{1 + \delta^t}$$

**FET-114-07: Capital Allocation in Financial Firms**

$$p.116 \quad \begin{aligned} NPV &= (1 - d)V\{S^+\} - (C - \mu) - (1 + m)V\{S^-\} \\ &= \mu - (dV\{S^+\} + mV\{S^-\}) \\ V\{S^+\} &= \frac{\sigma(n(z)+zN(z))}{(1+r)} \\ V\{S^-\} &= \frac{\sigma(n(z)-zN(-z))}{(1+r)} \\ z &= C(1 + r)/\sigma \end{aligned}$$

**FET-151-08: Babbel and Merrill, Real and Illusory Value Creation by Insurance Companies**

$$(2) \quad V(E) = F + V(A_T) - PV(L) + O$$

**Smith: Investor & Management Expectations of the Return on Equity Measure vs. Some Basic Truths of Financial Accounting**

$$E_t - EV_t = \sum_{x=1}^t [(ROE_x - IRR)E_{x-1}(1 + IRR)^{t-x}]$$